Electromagnetics Code Maintenance

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LLNL Engineering's EMSolve code is a 3-D, parallel, finite element code for solving Maxwell's equations. EMSolve has modules for electrostatics, magnetostatics, eigenvalues, eddy currents, and wave propagation. The code has been used to support NIF, the Weapons Program, and Global Security. In addition, EMSolve results have appeared in approximately 30 peer-reviewed publications. The purpose of this project is to enhance, verify, document, and maintain the EMSolve suite of computational electromagnetics codes.

Project Goals

The goal for FY2009 was to incorporate sensitivity analysis and uncertainty quantification into EMSolve in collaboration with Ohio State University. In finite element codes such as EMSolve, the geometry, material properties, and source terms are considered *input*, and field values (*e.g.*, electric field, magnetic field, and current density) are considered *output*. In virtually all engineering applications there is some uncertainty in input, and quantifying that effect upon output is referred to as uncertainty quantification. A key step in uncertainty quantification is sensitivity analysis, defined by computing the differential change in output due to a differential change in the input.

Relevance to LLNL Mission

EMSolve can perform electromagnetic analyses that cannot be undertaken with commercial codes. This allows better support for programs, and EMSolve's unique capability allows LLNL to be better positioned to assist other government agencies through work-for-others projects.

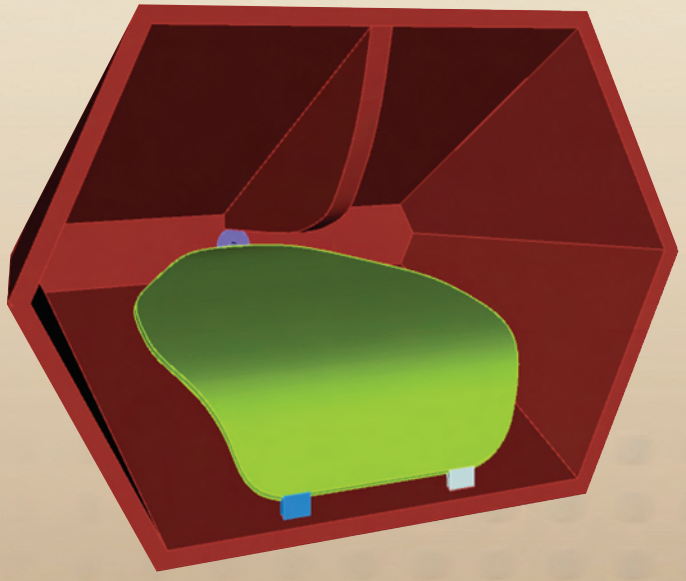

Figure 1. CAD model of an ultra-wideband antenna. This antenna is currently being used for several radar projects. The two small tabs are resistors, and the sensitivity of the radiated power with respect to these resistors can be determined using the Adjoint Method.

FY2009 Accomplishments and Results

We investigated several approaches for sensitivity analysis and determined that a good starting point is the Adjoint Method. For clarity of explanation we will write the frequency domain finite element representation of Maxwell's Equations as **Au = b**, where **b** is the source vector, **A** is the discretized Helmholtz Equation, and **u** is the solution vector. We have a set of parameters **p** = $\{p_1, p_2, \ldots, p_n\}$ that affect the solution vector **u**, and we have a response functional **R (p,u)** that is an engineering metric of interest, *e.g.*, total current or input impedance. Sensitivity analysis consists of computing

$$\nabla R = \left(\frac{dR}{dp_1}, \frac{dR}{dp_2}, \ldots, \frac{dR}{dp_N} \right)$$

$$\text{where } \frac{dR}{dp_i} = \frac{\partial R}{\partial p_i} + \frac{\partial R}{\partial u} \frac{\partial u}{\partial p_i}$$

We assume that the engineering metric of interest is linear; *i.e.*, **R = c u**. The sensitivity equation is then

$$\frac{dR}{dp_i} = c'u + cA^{-1}(b' - A'u)$$

where the prime denotes differentiation with respect to the parameter, **p**. The key step in the Adjoint Method is to introduce an adjoint variable, ψ, defined by the equation

$$A^H \Psi = c$$

The second term in the sensitivity equation becomes

$$cA^{-1}(b' - A'u) = \psi(b' - A'u),$$

the net effect being that to compute the sensitivity of **R** with respect to **N** parameters requires only two linear solves, one for **u** and one for ψ.

The Adjoint Method described above was implemented in the frequency domain module of EMSolve. For the time being the parameters **p** are restricted to be material properties, sensitivity with respect to geometry is significantly more complex and will be implemented next year.

As an example consider the ultra-wideband antenna illustrated in Fig. 1. The aperture is approximately 3.2 by 2.2 in. The two small tabs below the tongue are 100-Ω resistors. This antenna is designed to operate from 2 GHz to 5 GHz. We used the Adjoint Method to compute the sensitivity of the antenna-radiated power with respect to the conductivity and dielectric constant of the resistors.

The interesting sensitivity results are shown in Fig. 2. First, the sensitivity is low, meaning it is not necessary to fabricate precision resistors. While variability in radiated power between different antennas is experimentally observed, this sensitivity analysis shows that observed variability is not due to variation in resistors. Second, the sensitivity has the same sign for all frequencies, meaning the radiated power can be increased across the entire frequency band by reducing the value of the resistors. Third, the sensitivity with respect to the dielectric constant also has constant sign, thus adding a capacitor will increase the radiated power across the entire frequency band. This example shows how sensitivity analysis can be used both for assessing the effect of manufacturing variability and also for system design optimization.

Related References

Cacui, D., Sensitivity and Uncertainty Analysis, Volume 1, CRC Press, 2003.

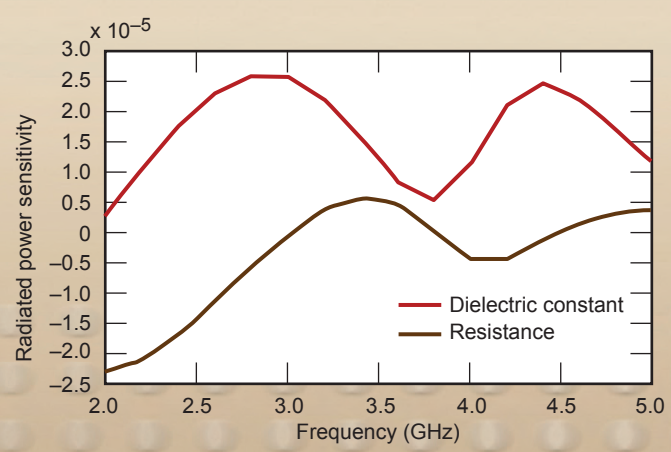

Figure 2. Sensitivity of radiated power with respect to resistance and capacitance of the two small tabs. This result can be used to optimize the antenna design and to determine acceptable manufacturing specifications.